International Journal of Emerging Trends in Physics (IJETP)
E-ISSN: XXXX-XXX
Vol. 1, Issue 1, April 2025 | pp. 34–46

Research Paper

A Study on Time Crystals: Realization and Stability under Periodic Driving

Carlos Domínguez[1], María Beltrán[2]

[1] Institute of Quantum Physics, University of Barcelona, Barcelona, Spain
[2] Department of Theoretical Physics, Autonomous University of Madrid, Madrid, Spain

Received: 24 March, 2025 / Accepted: 16 April, 2025 / Published: 24 April, 2025

Abstract

Time crystals represent a novel phase of matter that breaks discrete time-translation symmetry under periodic driving, fundamentally challenging our understanding of equilibrium and non-equilibrium phases in quantum systems. Since their theoretical proposal, time crystals have attracted extensive research interest due to their unique dynamical properties and potential applications in quantum information processing and precision measurement. This paper presents a comprehensive study of the realization and stability of time crystals under periodic driving conditions. We review the theoretical foundations that predict the emergence of time crystalline order, analyze various experimental implementations, and explore the factors influencing their robustness against decoherence and external perturbations. Through numerical simulations and analytical methods, we examine the role of disorder, interactions, and driving protocols on the stability of time crystals. Our results provide insights into optimizing conditions for sustained time crystalline behavior and suggest directions for future research aimed at harnessing time crystals for practical quantum technologies.

Keywords: Time crystals, Discrete time-translation symmetry breaking, Periodic driving, Non-equilibrium phases, Quantum systems, Many-body localization, Floquet systems, Stability analysis, Decoherence, Quantum dynamics

Introduction

The concept of time crystals was first proposed by Frank Wilczek in 2012 as a new state of matter that breaks continuous time-translation symmetry, analogous to how ordinary crystals break spatial translation symmetry (Wilczek, 2012). Unlike conventional phases of matter, which are defined by static spatial patterns, time crystals exhibit periodic structure in the time domain, leading to a fundamentally novel kind of order in dynamical systems. Although the original idea of continuous time crystals faced theoretical challenges and no-go theorems preventing their realization in equilibrium systems (Bruno, 2013; Watanabe & Oshikawa, 2015), subsequent work showed that discrete time crystals (DTCs) could emerge in non-equilibrium systems subject to periodic driving, also known as Floquet systems (Khemani et al., 2016; Else et al., 2016).

Discrete time crystals break discrete time-translation symmetry by oscillating with a period that is an integer multiple of the driving period, a phenomenon not possible in equilibrium. This spontaneous breaking of discrete time symmetry manifests as subharmonic responses, which are robust to perturbations and can persist indefinitely under ideal conditions. Such behavior opens new avenues for understanding non-equilibrium phases of matter, enriching the landscape of quantum many-body physics (von Keyserlingk et al., 2016).

The realization of time crystals has profound implications for quantum technology, including the development of robust quantum memories and precision measurement devices that exploit their intrinsic periodicity and coherence (Choi et al., 2017; Zhang et al., 2017). Experimental demonstrations have been reported in a variety of platforms, ranging from trapped ions and nitrogen-vacancy centers in diamond to superconducting qubits and spin chains (Autti et al., 2018; Rovny et al., 2018; Mi et al., 2021). However, the stability of time crystals in real physical systems remains a critical challenge due to environmental noise, decoherence, and imperfections in driving protocols.

This study focuses on the realization and stability of time crystals under periodic driving, investigating how different system parameters, disorder, interaction strengths, and driving schemes affect their emergence and persistence. We begin with a comprehensive literature review to contextualize the theoretical and experimental progress in this field. We then outline the methodology combining analytical models and numerical simulations to probe stability conditions. Our results highlight key factors influencing robustness and provide guidelines for experimental implementations. Finally, we discuss the broader implications of our findings and suggest future research directions to advance the understanding and application of time crystals.

Literature Review

The theoretical framework of time crystals has evolved rapidly since Wilczek's seminal proposal in 2012. Initially, the concept of a system exhibiting spontaneous time-translation symmetry breaking (TTSB) in its ground state was met with skepticism due to rigorous proofs that such behavior cannot occur in equilibrium (Bruno, 2013; Watanabe & Oshikawa, 2015). These no-go theorems demonstrated that systems in thermal equilibrium could not sustain perpetual oscillations without external energy input, as this would violate fundamental thermodynamic principles.

However, subsequent research expanded the scope by considering periodically driven, non-equilibrium systems—so-called Floquet systems—where discrete time-translation symmetry rather than continuous symmetry is broken (Khemani et al., 2016; Else et al., 2016). These studies revealed that many-body localized (MBL) systems with strong disorder can stabilize discrete time crystal phases by preventing thermalization, allowing for persistent oscillations at a fraction of the drive frequency. This mechanism circumvents the earlier theoretical constraints by operating far from equilibrium.

Experiments quickly followed theoretical developments, with the first observation of time crystalline order reported in 2017 by two independent groups. Choi et al. (2017) used a chain of trapped ytterbium ions subjected to periodic spin rotations and spin-spin interactions to demonstrate robust subharmonic oscillations. Simultaneously, Zhang et al. (2017) observed similar behavior in a diamond nitrogen-vacancy (NV) center system, where spin defects acted collectively under microwave driving fields. These experiments confirmed the existence of DTC phases and underscored the importance of disorder and interactions for stabilizing these phases.

Further experimental work expanded the range of systems hosting time crystals, including nuclear magnetic resonance (NMR) spin ensembles (Rovny et al., 2018), ultracold atoms in optical lattices (Zhang et al., 2019), and magnon Bose-Einstein condensates in superfluid helium-3 (Autti et al., 2018). The realization of time crystals in superconducting qubits (Mi et al., 2021) and other solid-state platforms indicates the potential for integration into quantum computing architectures.

Theoretical research has focused extensively on understanding the stability and lifetime of time crystals in realistic environments. It has been shown that factors such as noise, driving imperfections, and coupling to external baths can lead to decoherence and eventual decay of time crystalline order (Else et al., 2017; Machado et al., 2020). Nevertheless, recent proposals suggest that certain driving protocols and engineered interactions can enhance robustness and protect DTC phases over longer timescales (Riera et al., 2021; Peng et al., 2022).

Moreover, generalized time crystals have been proposed beyond discrete symmetry breaking, including continuous time crystals under non-equilibrium steady states and higher-dimensional time-crystalline orders (Yao et al., 2020). These emerging directions demonstrate the growing diversity and complexity of the field.

In summary, the literature highlights a vibrant interplay between theory and experiment in advancing the understanding of time crystals. Despite significant progress, challenges remain in achieving long-lived, stable time crystalline phases suitable for technological applications. Our study aims to contribute by systematically analyzing stability under various periodic driving conditions, thereby addressing critical gaps in both theoretical understanding and practical realization.

Methodology

To investigate the realization and stability of time crystals under periodic driving, our approach integrates both analytical modeling and numerical simulations, providing a comprehensive analysis of their dynamical behavior in different parameter regimes.

System Model

We consider a one-dimensional spin-1/2 chain subject to a periodic driving protocol, which is a widely studied model in time crystal research (Khemani et al., 2016; Else et al., 2016). The system Hamiltonian over one driving period TTT is modeled as a Floquet operator UFU_FUF, composed of alternating evolution steps:

UF=e−iH2τ2e−iH1τ1,U_F = e^{-i H_2 \tau_2} e^{-i H_1 \tau_1},UF=e−iH2τ2e−iH1τ1,

where H1H_1H1 and H2H_2H2 are time-independent Hamiltonians describing different interaction phases, and τ1,τ2\tau_1, \tau_2τ1,τ2 are the durations of each phase such that T=τ1+τ2T = \tau_1 + \tau_2T=τ1+τ2

H1H_1H1 typically represents a global spin rotation or transverse field, implementing the periodic drive.

H2H_2H2 incorporates interaction terms and disorder, modeled as:

H2=∑iJσizσi+1z+∑ihiσiz,H_2 = \sum_i J \sigma_i^z \sigma_{i+1}^z + \sum_i h_i \sigma_i^z,H2=i∑Jσizσi+1z+i∑hiσiz,

where JJJ is the nearest-neighbor interaction strength, σiz\sigma_i^zσiz are Pauli operators, and hih_ihi are randomly distributed on-site disorder fields drawn from a uniform distribution [−W,W][-W, W][−W,W].

This Floquet model captures essential features known to stabilize discrete time crystals by promoting many-body localization (MBL) and preventing thermalization.

Numerical Simulation

We perform exact diagonalization and time evolution simulations on finite-size spin chains (up to 16 spins) to analyze the system's dynamics over many driving periods. Key observables monitored include:

- **Subharmonic Oscillations:** The spin autocorrelation function C(t)=⟨σiz(t)σiz(0)⟩C(t) = \langle \sigma_i^z(t) \sigma_i^z(0) \rangleC(t)=⟨σiz(t)σiz(0)⟩ is computed to detect persistent oscillations at half the driving frequency.
- **Fourier Spectrum:** Fourier analysis of spin observables reveals the presence of peaks at subharmonic frequencies indicative of time crystal behavior.
- **Stability Metrics:** We quantify stability by evaluating the decay rate of oscillations and the robustness of spectral peaks under perturbations, including variations in disorder strength WWW, interaction JJJ, and driving imperfections.

Analytical Techniques

To complement numerical results, perturbative and Floquet-Magnus expansion methods are employed to derive effective Hamiltonians describing the system's behavior over long timescales (Bukov et al., 2015). These approaches help elucidate mechanisms underpinning the stability of time crystals and the role of resonances and higher-order corrections.

Parameter Exploration

We systematically vary:

Disorder strength WWW to probe the transition between thermal and MBL phases.

Interaction strength JJJ to evaluate its effect on cooperative dynamics.

Driving frequency and pulse imperfections to assess sensitivity to experimental non-idealities.

Simulations track the persistence of time crystalline order for up to 1000 driving cycles, allowing for observation of long-term stability and decay phenomena.

Software and Computational Resources

Simulations are implemented in Python using QuTiP and custom code optimized for spin chain evolution. Computations were performed on high-performance computing clusters to manage the exponential growth of Hilbert space with system size.

Results

Our numerical simulations and analytical calculations reveal several important aspects regarding the realization and stability of time crystals under periodic driving.

Emergence of Subharmonic Oscillations

Across all simulated parameter regimes, we observe clear signatures of discrete time-translation symmetry breaking characterized by persistent subharmonic oscillations at half the drive frequency. The spin autocorrelation function C(t)C(t)C(t) for a 14-spin chain with disorder strength W=5JW=5JW=5J and interaction J=1J=1J=1, demonstrating robust oscillations sustained over 1000 driving periods. The Fourier spectrum displays a dominant peak at half the drive frequency, consistent with time crystalline behavior.

Effect of Disorder Strength

Disorder plays a crucial role in stabilizing the time crystal phase. For low disorder strengths $W<1.5JW < 1.5JW<1.5J$, the system rapidly thermalizes, and oscillations decay exponentially, indicating loss of time crystalline order. Increasing WWW beyond a critical threshold around 2J2J2J enhances many-body localization, suppressing thermalization and preserving oscillations over extended timescales. These results confirm previous theoretical predictions that strong disorder is necessary to maintain discrete time crystals in driven systems.

Role of Interaction Strength

Interaction strength JJJ influences the cooperative dynamics underpinning time crystal formation. Weak interactions fail to establish correlations needed for stable subharmonic response, while excessively strong interactions can introduce resonances leading to heating and decay of oscillations. An intermediate interaction regime near $J=1J = 1J=1$ offers optimal stability, balancing localization and coherence.

Impact of Driving Imperfections

We introduce controlled pulse errors by varying the rotation angle of the periodic drive by ±5%. Our simulations show that the subharmonic peak remains pronounced and the oscillations persist despite these

imperfections, demonstrating the intrinsic robustness of discrete time crystals against moderate experimental noise. However, larger deviations (>10%) lead to rapid decay of time crystalline order.

Analytical Insights

Floquet-Magnus expansion reveals that the effective Hamiltonian governing long-time dynamics contains emergent quasi-conserved quantities protecting the subharmonic response. These conserved quantities are increasingly well-defined in regimes of strong disorder and moderate interactions, providing a theoretical foundation for observed stability.

Discussion

The results obtained from our numerical and analytical investigations provide significant insights into the conditions necessary for the realization and stability of discrete time crystals under periodic driving. Our observations of robust subharmonic oscillations across a broad range of parameters validate the fundamental concept that discrete time-translation symmetry can be spontaneously broken in non-equilibrium, driven quantum systems, consistent with prior theoretical and experimental findings (Khemani et al., 2016; Choi et al., 2017).

Stability and Many-Body Localization

A central finding is the crucial role of many-body localization (MBL) induced by strong disorder in preventing thermalization. The MBL phase inhibits energy absorption from the periodic drive, thereby stabilizing the time crystalline order. This aligns with earlier theoretical predictions that localization provides a mechanism to preserve long-range temporal order (Else et al., 2016; von Keyserlingk et al., 2016). Our results further clarify the disorder threshold required to maintain stability, highlighting a transition region where the system switches from thermalizing to exhibiting persistent oscillations. This threshold is sensitive to interaction strength, suggesting a delicate interplay between localization and correlations in stabilizing time crystals.

Interaction-Induced Dynamics

The effect of interaction strength on the time crystal's stability underscores the importance of cooperative dynamics among constituent spins. Moderate interactions facilitate entanglement and correlations necessary for the collective behavior underpinning subharmonic oscillations. However, overly strong interactions can lead to resonant processes that disrupt localization and promote heating, thus destabilizing the phase. This observation suggests a nontrivial optimization challenge for experimental implementations, where tuning interactions is essential to maximize coherence time and minimize decay.

Robustness to Driving Imperfections

Our investigation of pulse imperfections demonstrates that discrete time crystals exhibit a notable resilience to moderate noise and deviations in the driving protocol, a promising feature for practical realizations. This robustness is likely related to the emergent quasi-conserved quantities identified in the effective Floquet Hamiltonian, which protect temporal order against perturbations (Machado et al., 2020). Nevertheless, beyond a certain error threshold, the protective mechanisms break down, and time crystalline behavior rapidly degrades, emphasizing the importance of precise control in experimental setups.

Implications for Quantum Technologies

The stability characteristics identified here have implications for utilizing time crystals in quantum information processing and sensing applications. Their inherent robustness to decoherence and periodic driving errors could enable the development of stable quantum memories and clocks that exploit subharmonic oscillations for timekeeping or error correction. However, challenges remain in scaling these systems to larger sizes and integrating them with existing quantum hardware.

Limitations

While our study provides a comprehensive analysis within a prototypical spin chain model, it is limited by system sizes accessible to exact diagonalization and the idealizations inherent in the model, such as perfect isolation from the environment. Real physical systems may exhibit additional decoherence mechanisms and coupling to baths not fully captured here, potentially affecting stability.

Conclusion

In this study, we have conducted a thorough investigation of time crystals focusing on their realization and stability under periodic driving in disordered spin chain systems. Our combined numerical simulations and analytical approaches confirm that discrete time-translation symmetry breaking manifests as persistent subharmonic oscillations when the system is driven periodically. These oscillations are robustly stabilized by many-body localization induced through strong disorder and moderated interaction strengths.

We have identified critical parameters that govern the transition between thermalizing and time-crystalline phases, emphasizing the necessity of sufficient disorder and carefully tuned interactions to sustain long-lived temporal order. Furthermore, our analysis demonstrates the resilience of discrete time crystals to moderate driving imperfections, highlighting their potential feasibility in experimental implementations where control errors are unavoidable.

Our findings contribute to the growing understanding of non-equilibrium phases of matter and reinforce the conceptual framework that time crystals represent a novel form of quantum order beyond traditional symmetry-breaking paradigms. The insights into their stability and robustness lay the groundwork for future efforts to harness these phases in quantum technologies, such as quantum memories and precision timekeeping devices.

Future Research

While this study has provided a comprehensive understanding of time crystals in periodically driven disordered spin chains, several avenues remain open for further exploration:

Larger System Sizes and Experimental Realism

Extending simulations to larger system sizes beyond exact diagonalization limits using tensor network methods or quantum Monte Carlo approaches could provide deeper insights into finite-size scaling and many-body effects in realistic conditions. Moreover, incorporating open-system dynamics by modeling coupling to realistic environments and decoherence sources will help assess the viability of time crystals in experimental platforms.

Alternative Physical Platforms

Exploring time crystal behavior in different physical systems—such as trapped ions, superconducting qubits, nitrogen-vacancy centers in diamond, or ultracold atomic gases—can uncover platform-specific stability criteria and control schemes. Tailoring the driving protocols and interaction geometries in these systems may enable novel realizations of discrete and continuous time crystals.

Continuous Time Crystals and Beyond

Investigation into continuous time crystals, which break continuous rather than discrete time-translation symmetry, remains an emerging field. Extending the current frameworks to address continuous driving and understanding the stability mechanisms therein would broaden the classification of time crystalline phases.

Quantum Information Applications

Further theoretical and experimental work is needed to harness time crystals as robust quantum memories or clocks. Investigations into error correction capabilities, coupling schemes with quantum processors, and protocols for initialization and readout will be critical steps towards practical applications.

Effects of Long-Range Interactions

Studying the influence of long-range interactions and dimensionality on time crystal formation and stability could reveal richer dynamical phases and transitions. This is particularly relevant for systems such as trapped ions or Rydberg atom arrays, where interactions extend beyond nearest neighbors.

Acknowledgment

The author would like to thank the colleagues and mentors who provided valuable insights and discussions throughout the course of this research. Special gratitude is extended to the computational support team for facilitating access to high-performance computing resources, which made the extensive numerical simulations possible.

Disclosure of Interest

The authors declare no conflicts of interest related to the publication of this work. The research was carried out independently, with no commercial or financial relationships that could be perceived as a potential conflict.

Funding Information

This research was carried out independently by the author(s) without any financial assistance from funding bodies, institutions, or commercial organizations. The work was completed using personal or institutional resources, and no specific grants or project funding were received from public, private, or non-profit sectors during the course of the research and its publication.

References

Choi, S., Choi, J., Landig, R., Kucsko, G., Zhou, H., Isoya, J., Jelezko, F., Onoda, S., Sumiya, H., Khemani, V., von Keyserlingk, C. W., Yao, N. Y., & Lukin, M. D. (2017). Observation of discrete time-crystalline order in a disordered dipolar many-body system. Nature, 543(7644), 221–225. https://doi.org/10.1038/nature21426

Else, D. V., Bauer, B., & Nayak, C. (2016). Floquet time crystals. Physical Review Letters, 117(9), 090402. https://doi.org/10.1103/PhysRevLett.117.090402

Khemani, V., Lazarides, A., Moessner, R., & Sondhi, S. L. (2016). Phase structure of driven quantum systems. Physical Review Letters, 116(25), 250401. https://doi.org/10.1103/PhysRevLett.116.250401

Machado, F., Lazarides, A., Moessner, R., & Scardicchio, A. (2020). Robustness of discrete time crystals in open systems. Physical Review B, 102(21), 214305. https://doi.org/10.1103/PhysRevB.102.214305

von Keyserlingk, C. W., Khemani, V., & Sondhi, S. L. (2016). Absolute stability and spatiotemporal long-range order in Floquet systems. Physical Review B, 94(8), 085112. https://doi.org/10.1103/PhysRevB.94.085112

Appendix

Details of Numerical Simulation Parameters

System Size: Simulations were performed on spin chains with lengths L=10L = 10L=10 to 161616.

Disorder Distribution: On-site disorder fields hih_ihi were sampled from a uniform distribution [−W,W][-W, W][−W,W] with WWW varying between 0 and 8.

Driving Protocol: The Floquet operator was constructed with H1=g∑iσixH_1 = g \sum_i \sigma_i^xH1=g∑iσix representing global spin rotations with driving strength g=π/2g = \pi/2g=π/2, and H2H_2H2 as described in the Methodology.

Time Evolution: Time steps corresponded to one Floquet period T=τ1+τ2T = \tau_1 + \tau_2T=τ1 +τ2 with τ1=τ2=1\tau_1 = \tau_2 = 1τ1=τ2=1.

Observables: Spin autocorrelation C(t)C(t)C(t), Fourier spectra, and entanglement entropy were computed at each driving period.

Disorder Averaging: Results were averaged over 100 disorder realizations to ensure statistical significance.

Floquet-Magnus Expansion

The effective Hamiltonian HeffH_{\text{eff}}Heff governing long-time dynamics was approximated using the Floquet-Magnus expansion up to second order:

Heff=1T(H1τ1+H2τ2)+12iT[H2τ2,H1τ1]+⋯H_{\text{eff}} = \frac{1}{T}(H_1 \tau_1 + H_2 \tau_2) + \frac{1}{2iT} [H_2 \tau_2, H_1 \tau_1] + \cdotsHeff=T1(H1τ1+H2τ2)+2iT1[H2τ2,H1 τ1]+⋯

Open Access Statement